

Massive Tensor Multiplets in $N = 1$ Supersymmetry¹

Jan Louis and Waldemar Schulgin

II. Institut für Theoretische Physik

Universität Hamburg

Luruper Chaussee 149

D-22761 Hamburg, Germany

jan.louis@desy.de, waldemar.schulgin@desy.de

ABSTRACT

We derive the action for a massive tensor multiplet coupled to chiral and vector multiplets as it can appear in orientifold compactifications of type IIB string theory. We compute the potential of the theory and show its consistency with the corresponding Kaluza-Klein reduction of $N = 1$ orientifold compactifications. The potential contains an explicit mass term for the scalar in the tensor multiplet which does not arise from eliminating an auxiliary field. A dual action with an additional massive vector multiplet is derived at the level of superfields.

October 2004

¹Work supported by: DFG – The German Science Foundation, European RTN Program HPRN-CT-2000-00148 and the DAAD – the German Academic Exchange Service.

1 Introduction

Consistent string theories as we know them today come equipped with some amount of supersymmetry. As a consequence there is a continuous vacuum degeneracy parameterized by the vacuum expectation values of scalar fields called moduli. In order to make contact with particle physics it is necessary to break the supersymmetry and lift the vacuum degeneracy. It has been realized that compactifications of string theory with non-trivial background fluxes generically generate a potential for the moduli scalars and spontaneously break supersymmetry [1, 2, 3, 4, 5, 6] possibly with a stable local de Sitter ground state [7, 8].

In the low energy supergravity the fluxes appear as gauge or mass parameters and turn an ordinary supergravity into a gauged or massive supergravity. In the latter case the fluxes induce mass terms for some of the p -form gauge potentials which are present in string theories [9, 10, 11]. For example, in $N = 2$ theories in four space-time dimensions massive two-forms appear in compactification of type II string theory on Calabi-Yau threefolds when both electric and magnetic three-form fluxes are turned on [11]. The corresponding $N = 2$ supergravity theories have been constructed recently in [12]. The same phenomenon can also be observed in $N = 1$ compactifications of type IIB on Calabi-Yau orientifolds with $O5$ - or $O9$ -planes [13]. Surprisingly, in terms of the low energy $N = 1$ effective supergravity a massive two-form has not been discussed in considerable detail in the literature [14, 15, 16].² The purpose of this paper is to (partially) close this gap and study the superspace action proposed in [13].

In $N = 1$ supersymmetry an antisymmetric tensor resides in a chiral spinor superfield Φ_α while its three-form field strength is a member of a linear Multiplet L [14]. When the antisymmetric tensor is massless the action is expressed in terms of L only and well known [14, 15, 16, 19, 20, 21, 22]. However, when the antisymmetric tensor is massive the action explicitly features Φ_α and gauge invariance requires a Stückelberg-type coupling to at least one vector multiplet V . In this paper we derive the supersymmetric action for such a massive antisymmetric tensor coupled to n_V vector multiplets V^A , $A = 1, \dots, n_V$. We additionally allow for the possibility that the mass depends on an arbitrary number of chiral multiplets N^i , $i = 1, \dots, n_c$. This situation is partly motivated by the results of the orientifolds compactifications and we explicitly rederive the supersymmetric effective action obtained in [13] from a Kaluza-Klein reduction. Curiously the potential is not determined solely by the auxiliary fields but contains an explicit mass term for the scalar in the tensor multiplet.

This paper is organized as follows. In section 2.1 we discuss the spinor superfield Φ_α , its field strength L and their gauge transformation properties. In section 2.2 we write down the most general gauge invariant superspace action for Φ_α including a Stückelberg-type mass term and explicitly give its corresponding component form. We discuss in detail the resulting potential and show that it does not have a ‘standard’ $N = 1$ form. In section 3 we perform the duality transformation in superfields and rewrite the action in terms of $n_V - 1$ massless and one massive vector multiplet. An expanded version of this work can be found in [23].

²While this manuscript was being prepared we received a preprint [17] which discusses the same topic. Some phenomenological applications have recently been discussed in [18].

2 The action of a massive antisymmetric tensor

2.1 The supermultiplets

Let us start by introducing the supermultiplets which contain an antisymmetric tensor B_{mn} and its field strength H_{mnp} . The latter resides in a linear superfield L which is real and obeys the additional constraint

$$D^2 L = \bar{D}^2 L = 0 , \quad (2.1)$$

where D_α is the superspace covariant derivative.³ In components it contains a real scalar C , the field strength of an antisymmetric tensor $H_{mnp} = \partial_{[m} B_{np]}$ and a Weyl fermion η . Its θ -expansion is obtained by solving (2.1) and reads

$$L = C + \theta\eta + \bar{\theta}\bar{\eta} + \frac{1}{2}\theta\sigma^m\bar{\theta}\epsilon_{mnpq}H^{npq} - \frac{i}{2}\theta\theta\bar{\theta}\bar{\sigma}^m\partial_m\eta - \frac{i}{2}\bar{\theta}\bar{\theta}\theta\sigma^m\partial_m\bar{\eta} - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\square C . \quad (2.2)$$

The antisymmetric tensor B_{mn} itself is contained in a chiral spinor multiplet Φ_α defined via

$$L = \frac{1}{2}(D^\alpha\Phi_\alpha + \bar{D}_{\dot{\alpha}}\bar{\Phi}^{\dot{\alpha}}) , \quad \bar{D}_{\dot{\beta}}\Phi_\alpha = 0 . \quad (2.3)$$

With this definition the constraints (2.1) are solved identically. Furthermore, due to the identity $D^\alpha\bar{D}^2D_\alpha = \bar{D}_{\dot{\alpha}}D^2\bar{D}^{\dot{\alpha}}$, L is left invariant by the gauge transformation

$$\Phi_\alpha \rightarrow \Phi_\alpha + \frac{i}{8}\bar{D}^2D_\alpha\Lambda , \quad (2.4)$$

where Λ is a real superfield.

Φ_α contains off shell eight bosonic plus eight fermionic degrees of freedom, four of which are rendered unphysical by the gauge invariance (2.4). This can be seen as follows. A chiral spinor superfield Φ_α has a generic θ -expansion

$$\Phi_\alpha = \chi_\alpha - \left(\frac{\delta_\alpha{}^\gamma(C + iE)}{2} + \frac{1}{4}(\sigma^m\bar{\sigma}^n)_\alpha{}^\gamma B_{mn} \right) \theta_\gamma + \theta\theta \left(\eta_\alpha + i\sigma_{\alpha\dot{\alpha}}{}^m \partial_m \bar{\chi}^{\dot{\alpha}} \right) . \quad (2.5)$$

Since $\bar{D}^2D_\alpha\Lambda$ is chiral it enjoys the expansion

$$\frac{i}{8}\bar{D}^2D_\alpha\Lambda = -\frac{1}{2}\lambda_\alpha - \left(\delta_\alpha{}^\beta \frac{iD}{2} + \frac{1}{4}(\sigma^m\bar{\sigma}^n)_\alpha{}^\beta (\partial_m\Lambda_n - \partial_n\Lambda_m) \right) \theta_\beta - \frac{i}{2}\theta\theta\sigma_{\alpha\dot{\alpha}}{}^m \partial_m \bar{\lambda}^{\dot{\alpha}} . \quad (2.6)$$

Comparing (2.5) and (2.6) we see that the component fields χ_α and E of Φ_α can be gauged away leaving only the physical degrees of freedom which are C , B_{mn} and η in the component expansion of Φ_α

$$\Phi_\alpha = -\frac{1}{2}\theta_\gamma(\delta_\alpha{}^\gamma C + \frac{1}{2}(\sigma^m\bar{\sigma}^n)_\alpha{}^\gamma B_{mn}) + \theta\theta\eta_\alpha . \quad (2.7)$$

In this gauge the left over gauge invariance is the standard two-form gauge invariance

$$B_{mn} \rightarrow B_{mn} + \partial_m\Lambda_n - \partial_n\Lambda_m , \quad \eta \rightarrow \eta , \quad C \rightarrow C . \quad (2.8)$$

³We are following the conventions of [24] and abbreviate $D^2 = D^\alpha D_\alpha$ and $\bar{D}^2 = \bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}$.

2.2 Action of a massive tensor multiplet

The next step is the construction of a gauge invariant Lagrangian for a massive antisymmetric tensor. Let us start with the kinetic term which arises from [22]

$$\mathcal{L}_{\text{kin}} = - \int d^2\theta d^2\bar{\theta} K(L) , \quad (2.9)$$

where $K(L)$ is an arbitrary real function. Using (2.2) one finds the component action

$$\begin{aligned} \mathcal{L}_{\text{kin}} = & -\frac{1}{4}K'' \left(\partial_m C \partial^m C + i(\eta \sigma^m \partial_m \bar{\eta} + \bar{\eta} \bar{\sigma}^m \partial_m \eta) + \frac{3}{2} H_{mnp} H^{mnp} \right) \\ & -\frac{1}{8} K''' \eta \sigma^m \bar{\eta} \epsilon_{mnpq} H^{npq} - \frac{1}{48} K'''' \eta \eta \bar{\eta} \bar{\eta} , \end{aligned} \quad (2.10)$$

where $K'' = \partial_C^2 K$, $K''' = \partial_C^3 K$ etc. .

The mass term for B_{mn} arises from a chiral superspace integral which is quadratic in Φ_α and reads $\mathcal{L}_m \sim \int d^2\theta \Phi^\alpha \Phi_\alpha + \text{h.c.}$. Taken at face value such a mass term is not gauge invariant under the gauge transformation (2.4). However, by appropriately coupling the spinor superfield Φ_α to n_V Abelian vector multiplets V^A , $A = 1, \dots, n_V$ gauge invariant mass terms can be constructed. The couplings have to be such that the vector multiplets provide the necessary degrees of freedom for a massive B_{mn} to exist. Or in other words we need to employ a Stückelberg mechanism for B_{mn} [25].

The chiral field strengths of the vector multiplets are denoted by $W_\alpha^A = -\frac{1}{4}\bar{D}^2 D_\alpha V^A$ and have a component expansion [24]

$$W_\alpha^A = -i\lambda_\alpha^A + (\delta_\alpha^\beta D^A - \frac{i}{2}(\sigma^m \bar{\sigma}^n)_\alpha^\beta F_{mn}^A) \theta_\beta + \theta \theta \sigma_{\alpha\dot{\alpha}}^m \partial_m \bar{\lambda}^{\dot{\alpha}A} , \quad (2.11)$$

where $F_{mn}^A = \partial_m v_n^A - \partial_n v_m^A$ are the field strengths of n_V $U(1)$ gauge boson v_n^A . From the component form of the gauge transformation (2.8) we see that $\tilde{F}_{mn}^A := F_{mn}^A - m^A B_{mn}$ are gauge invariant combinations provided we assign to the gauge bosons v_n^A the transformation laws

$$v_n^A \rightarrow v_n^A + m^A \Lambda_n . \quad (2.12)$$

At this point the m^A are n_V parameters which, as we will see shortly, are related to the mass of B_{mn} . In terms of superfields the transformation law (2.12) translates into⁴

$$V^A \rightarrow V^A + m^A \Lambda , \quad W_\alpha^A \rightarrow W_\alpha^A - \frac{1}{4} m^A \bar{D}^2 D_\alpha \Lambda . \quad (2.13)$$

Physically the transformation laws (2.4), (2.12) and (2.13) state that one vector superfield is ‘pure gauge’ or in other words can be ‘eaten’ by the spinor superfield Φ_α . As we will see shortly one can remove one of the vector multiplets from the action by going to the equivalent of the unitary gauge. However, it turns out to be more convenient to keep all vectors and leave the action manifestly gauge invariant under the combined transformation (2.4) and (2.13). Such an invariant action we are going to construct next.

⁴Of course one also has the standard $U(1)$ gauge invariance $V^A \rightarrow V^A + \Sigma^A + \bar{\Sigma}^A$, $W_\alpha^A \rightarrow W_\alpha^A$ for chiral superfield parameters Σ^A .

We need to find all Lorentz-invariant expressions which are also singlets under the combined transformations given in (2.13) and (2.4). One immediately observes that $(W_\alpha^A - 2im^A\Phi_\alpha)$ is a gauge invariant combination of superfields. In order to find possible other combinations we make the Lorentz-invariant Ansatz

$$\mathcal{L}_m = \frac{1}{4} \int d^2\theta (f_{AB} W^A W^B + g_A \Phi W^A + h \Phi \Phi) + \text{h.c.} , \quad (2.14)$$

and determine the couplings f_{AB}, g_A and h by demanding \mathcal{L}_m to be invariant under (2.13) and (2.4) up to a total derivative. This determines

$$\mathcal{L}_m = \frac{1}{4} \int d^2\theta \left(f_{AB}(N)(W^A - 2im^A\Phi)(W^B - 2im^B\Phi) + 2e_A \Phi(W^A - im^A\Phi) \right) + \text{h.c.} . \quad (2.15)$$

The first term in the Lagrangian (2.15) contains the manifestly gauge invariant combination $(W_\alpha^A - 2im^A\Phi_\alpha)$ and thus the arbitrary coupling matrix $f_{AB}(N^i)$ can depend holomorphically on a set of n_c chiral superfields $N^i, i = 1, \dots, n_c$.⁵ The second term in (2.15) is only gauge invariant up to a total derivative and thus its couplings e_A have to be constants. The Lagrangian (2.15) is the first result of this paper in that, together with (2.9), it gives the most general Lagrangian of a massive spinor multiplet coupled to n_V vector and n_c chiral multiplets. This form of the action has also been proposed in [13].

Before we expand the action in components let us briefly discuss the limit $m^A = 0$ where (2.15) reduces to $\mathcal{L}_m = \frac{1}{4} \int d^2\theta (f_{AB} W^A W^B + 2e_A \Phi W^A) + \text{h.c.}$. The second term is known as a four-dimensional Green-Schwarz term and it is conventionally written in terms of a full superspace integral including L but not Φ_α [21, 22]. Indeed, using (2.3), the definition of W_α and partial integration one shows

$$\begin{aligned} e_A \int d^2\theta \Phi W^A + \text{h.c.} &= -\frac{1}{4} e_A \int d^2\theta \Phi \bar{D}^2 D V^A - \frac{1}{4} e_A \int d^2\bar{\theta} \bar{\Phi} D^2 \bar{D} V^A \\ &= e_A \int d^2\theta d^2\bar{\theta} (D\Phi + \bar{D}\bar{\Phi}) V^A = 2e_A \int d^2\theta d^2\bar{\theta} L V^A . \end{aligned}$$

Let us now expand the action (2.15) in components. Apart from using (2.2), (2.7), (2.11) we also need

$$N^i = A^i + \sqrt{2}\theta\chi^i + \theta\theta F^i , \quad (2.16)$$

$$f_{AB}(N) = f_{AB}(A) + \sqrt{2}\theta\chi^i \partial_i f_{AB} + \theta\theta (F^i \partial_i f_{AB} - \frac{1}{2} \chi^i \chi^j \partial_i \partial_j f_{AB}) .$$

Inserting (2.7), (2.11) and (2.16) into (2.15) we find

$$\begin{aligned} \mathcal{L}_m &= -\frac{1}{4} \text{Re} f_{AB} \check{F}_{mn}^A \check{F}^{Bmn} + \frac{1}{8} \text{Im} f_{AB} \epsilon^{mnp r} \check{F}_{mn}^A \check{F}_{pr}^B - \frac{1}{16} \epsilon^{mnp r} B_{mn} e_A (\check{F}_{pr}^A + F_{pr}^A) \\ &\quad + \frac{1}{2} \text{Re} f_{AB} D^A D^B - \frac{1}{2} C D^A (e_A + 2 \text{Im} f_{AB} m^B) - \frac{1}{2} \text{Re} f_{AB} m^A m^B C^2 \\ &\quad - \frac{i}{2} f_{AB} \lambda^A \sigma^m \partial_m \bar{\lambda}^B - \frac{i}{2} f_{AB}^* \bar{\lambda}^A \bar{\sigma}^m \partial_m \lambda^B \\ &\quad - \frac{1}{2} (ie_A + 2f_{AB} m^B) \eta \lambda^A - \frac{1}{2} (-ie_A + 2f_{AB}^* m^B) \bar{\eta} \bar{\lambda}^A + \dots , \end{aligned} \quad (2.17)$$

⁵Coupling the spinor superfield to chiral multiplets also requires a kinetic term for the N^i but since this term plays no role for the purpose of this paper we do not include it here.

where we abbreviated the gauge invariant combination as

$$\check{F}_{mn}^A = F_{mn}^A - m^A B_{mn} , \quad (2.18)$$

and where the ellipses denote terms proportional to $\partial_i f_{AB}$.⁶

The next step is to eliminate the auxiliary fields D^A by their equations of motion. Inserting the solution back into (2.17) we arrive at

$$\begin{aligned} \mathcal{L}_m = & -\frac{1}{4} \text{Re} f_{AB} \check{F}_{mn}^A \check{F}^{Bmn} + \frac{1}{8} \text{Im} f_{AB} \epsilon^{mnpq} \check{F}_{mn}^A \check{F}_{pq}^B - \frac{1}{16} \epsilon^{mnpq} B_{mn} e_A (\check{F}_{pq}^A + F_{pq}^A) - V \\ & - \frac{i}{2} f_{AB} \lambda^A \sigma^m \partial_m \bar{\lambda}^B - \frac{i}{2} f_{AB}^* \bar{\lambda}^A \bar{\sigma}^m \partial_m \lambda^B \\ & - \frac{1}{2} (ie_A + 2f_{AB} m^B) \eta \lambda^A - \frac{1}{2} (-ie_A + 2f_{AB}^* m^B) \bar{\eta} \bar{\lambda}^A + \dots , \end{aligned} \quad (2.19)$$

where the potential V is given by

$$V = \frac{1}{2} \text{Re} f_{AB} D^A D^B + \frac{1}{2} \text{Re} f_{AB} m^A m^B C^2 , \quad D^A = \frac{1}{2} \text{Re} f^{-1AB} (e_B + 2\text{Im} f_{BC} m^C) C . \quad (2.20)$$

We see that only part of the potential is generated by D -terms. In addition there is a contribution to the mass-term for the scalar C which does not arise from eliminating an auxiliary field. For $m^A = 0$ the mass term for C does come from a D -term and is part of the Green-Schwarz term discussed earlier. Explicitly inserting D^A we arrive at

$$\begin{aligned} V &= \frac{1}{8} \left((e_A + 2\text{Im} f_{AC} m^C) \text{Re} f^{-1AB} (e_B + 2\text{Im} f_{BD} m^D) + 4\text{Re} f_{AB} m^A m^B \right) C^2 \\ &= \frac{1}{8} \left((e_A - 2i f_{AC} m^C) \text{Re} f^{-1AB} (e_B + 2i \bar{f}_{BD} m^D) \right) C^2 . \end{aligned} \quad (2.21)$$

The potential (2.21) can be viewed as the $N = 1$ version of the $N = 2$ Taylor-Vafa potential given in [3] and rederived in [11]. Note that it does not follow from a superpotential but instead from a different chiral superspace integral (2.15). In [13] this potential has been derived by a Kaluza-Klein reduction of certain type IIB orientifolds. One purpose of this paper was to show how this potential arises from the superspace action (2.15).

Let us also discuss the mass terms of the antisymmetric tensor as they appear in the Lagrangian (2.19). They arise from the couplings including \check{F}_{mn} and displaying them separately one has (the analogous $N = 2$ expression can be found in [11])

$$\begin{aligned} \mathcal{L}_{\text{mB}} &= -\frac{1}{4} M^2 B_{mn} B^{mn} + \frac{1}{8} M_T^2 \epsilon^{mnpq} B_{mn} B_{pq} , \\ M^2 &= \text{Re} f_{AB} m^A m^B , \\ M_T^2 &= \text{Im} f_{AB} m^A m^B + \frac{1}{2} e_A m^A . \end{aligned} \quad (2.22)$$

For $m^A = 0$ both mass terms vanish and one is left with a massless antisymmetric tensor with a Green-Schwarz coupling of the form $\epsilon^{mnpq} F_{mn} B_{pq}$ as can be seen from (2.19).

⁶The complete action can be found in [23].

3 Dualities of an antisymmetric tensor

In this section we discuss the dual formulations of the antisymmetric tensor.⁷ We perform the duality transformation in superfields and then expand the dual action in components. Alternatively one can perform the duality transformation at the component level starting from (2.19). Of course, both methods give the same dual action in components and the details of the second route can be found in ref. [23].

We need to distinguish between the massless case $m^A = 0$ and the massive case $m^A \neq 0$. A massless antisymmetric tensor is dual to a scalar or at the level of superfields a linear multiplet is dual to a chiral multiplet. On the other hand a massive antisymmetric tensor is dual to a massive vector or at the superfield level a spinor superfield is dual to a massive vector multiplet. Let us discuss both cases in turn.

3.1 Pure Green-Schwarz term – massless linear multiplet

For $m^A = 0$ the Lagrangian can be expressed entirely in terms of L and reads

$$\begin{aligned}\mathcal{L} &= - \int d^2\theta d^2\bar{\theta} K(L) + \frac{1}{4} \int d^2\theta \left(f_{AB} W^A W^B + 2e_A \Phi W^A \right) + \text{h.c.} , \\ &= - \int d^2\theta d^2\bar{\theta} \left(K(L) - e_A L V^A \right) + \frac{1}{4} \int d^2\theta f_{AB} W^A W^B + \text{h.c.} ,\end{aligned}\quad (3.1)$$

where we used (2.16). In order to perform the duality transformation one treats L as a real (but not linear) superfield and adds the term [21, 22]

$$\delta\mathcal{L} = \int d^2\theta d^2\bar{\theta} L(S + \bar{S}) , \quad (3.2)$$

where S is a chiral superfield $\bar{D}_{\dot{\alpha}} S = 0$. The equations of motion for S and \bar{S} impose the constraint (2.1) and lead back to the action (3.1).⁸ The equation of motion for L on the other hand reads

$$-\partial_L K + e_A V^A + S + \bar{S} = 0 , \quad (3.3)$$

which expresses L in terms of the combination $e_A V^A + S + \bar{S}$. The precise relation depends on the specific form of $K(L)$. Inserting the solution of (3.3) into the sum of (3.1) and (3.2) one obtains

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} \hat{K}(e_A V^A + S + \bar{S}) + \frac{1}{4} \int d^2\theta f_{AB} W^A W^B + \text{h.c.} , \quad (3.4)$$

where $\hat{K} = -K + (e_A V^A + S + \bar{S})L$ is the Legendre transform of $K(L)$. The $U(1)$ gauge invariance of the vector multiplets is maintained as long as S transforms according to

$$V^A \rightarrow V^A + \Sigma^A + \bar{\Sigma}^A , \quad S \rightarrow S - e_A \Sigma^A , \quad (3.5)$$

⁷This has also been discussed recently in [26, 17].

⁸The equations of motion for S are found by representing S in terms of an unconstrained superfield X via $S = \bar{D}^2 X$ and varying the action with respect to X . (See [22] for more details.)

where Σ is a chiral superfield $\bar{D}_{\dot{\alpha}}\Sigma = 0$. We see that S can be absorbed into V^A rendering one linear combination of the V^A massive with a mass given by the second derivative of \hat{K} . Thus S is the (charged) Goldstone multiplet ‘eaten’ by a linear combination of the V^A .

In order to derive the component form of the action (3.4) we continue to expand V^A in a Wess-Zumino gauge with component fields $(v_m^A, \lambda_\alpha^A, D^A)$ as specified in (2.11) but include the components of S as physical fields with a θ -expansion $S = E + \sqrt{2}\theta\psi + \theta\theta F$. Inserted into (3.4) we arrive at

$$\begin{aligned}
\mathcal{L} = & \frac{1}{2}\hat{K}'e_AD^A + \frac{1}{2}\hat{K}''\left(2FF^* - \partial_m(\text{Re } E)\partial^m(\text{Re } E)\right. \\
& - \frac{1}{2}(e_Av_m^A + 2\partial_m(\text{Im } E))(e_Bv^{Bm} + 2\partial^m(\text{Im } E)) \\
& + i\sqrt{2}e_A(\psi\lambda^A - \bar{\psi}\bar{\lambda}^A) - i(\psi\sigma^m\partial_m\bar{\psi} + \bar{\psi}\bar{\sigma}^m\partial_m\psi)\Big) + \frac{1}{4}\hat{K}''''\psi\psi\bar{\psi}\bar{\psi} \quad (3.6) \\
& + \frac{1}{2}\hat{K}''' \left(-F^*\psi\psi - F\bar{\psi}\bar{\psi} + \bar{\psi}\bar{\sigma}^m\psi(e_Av_m^A + 2\partial_m(\text{Im } E))\right) \\
& - \frac{1}{4}\text{Re}f_{AB}F^{Amn}F_{mn}^B + \frac{1}{2}\text{Re}f_{AB}D^AD^B + \frac{1}{8}\text{Im}f_{AB}\epsilon^{mnp}rF_{mn}^AF_{pr}^B \\
& - \frac{i}{2}(f_{AB}\lambda^A\sigma^m\partial_m\bar{\lambda}^B - f_{AB}^*\partial_m\lambda^A\sigma^m\bar{\lambda}^B) + \dots,
\end{aligned}$$

where the ellipses once more denote terms proportional to $\partial_i f_{AB}$ and we abbreviated $\hat{K}' = \partial_{\text{Re } E}\hat{K}$, *etc.*. We see that the real scalar $\text{Im } E$ plays the role of the Goldstone boson which can be absorbed into the linear combination $e_Av_m^A$ by an appropriate gauge transformation (unitary gauge) leaving a mass term behind.

The auxiliary fields F and D^A can be eliminated by their equations of motions. Neglecting fermionic contributions we find

$$F = 0, \quad D^B = -\frac{1}{2}\hat{K}'e_A(\text{Re } f)^{-1AB}. \quad (3.7)$$

Inserted back into (3.6) and going to the unitary gauge the bosonic action reads

$$\begin{aligned}
\mathcal{L}_b = & -\frac{1}{4}\text{Re}f_{AB}F^{Amn}F_{mn}^B + \frac{1}{8}\text{Im}f_{AB}\epsilon^{mnp}rF_{mn}^AF_{pr}^B - \frac{1}{2}e_Ae_Bv_m^Av^{Bm} \\
& - \frac{1}{2}\hat{K}''\partial_m(\text{Re } E)\partial^m(\text{Re } E) - V, \quad (3.8)
\end{aligned}$$

where the potential is given by

$$V = \frac{1}{2}\text{Re}f_{AB}D^AD^B = \frac{1}{8}(\hat{K}')^2e_A(\text{Re } f)^{-1AB}e_B. \quad (3.9)$$

We see that this potential agrees with the dual potential given in (2.21) for $\hat{K}' = C$ which is just expressing the Legendre transformation between the dual variables discussed below (3.4). As a consequence also the kinetic terms of the scalars in (2.10) and (3.8) agree.

3.2 Duality for a massive tensor multiplet

Let us now discuss the situation when $m^A \neq 0$ and thus the antisymmetric is massive (c.f. (2.22)). In this case B_{mn} is dual to a massive vector or, at the level of superfields, Φ_α is dual to a massive vector multiplet.

We start from the (first order) Lagrangian [20]

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} \left(U(V^0) - V^0(D\Phi + \bar{D}\bar{\Phi}) \right) + \mathcal{L}_m, \quad (3.10)$$

where \mathcal{L}_m is given in (2.15). Φ_α continues to be a chiral spinor superfield while V^0 is a new real superfield in addition to the n_V vector fields V^A already present in \mathcal{L}_m . As we will see shortly V^0 will play the role of the dual massive vector multiplet. Note that (3.10) is not gauge invariant and therefore we cannot take V^0 in the WZ-gauge.

Let us first check that eliminating V^0 by its equations of motion leads us back to the action $\mathcal{L}_{\text{kin}} + \mathcal{L}_m$ given in (2.9) and (2.15) respectively. From (3.10) we determine the field equation of V^0 to be

$$\frac{\partial U}{\partial V^0} = 2L = D\Phi + \bar{D}\bar{\Phi}. \quad (3.11)$$

For appropriate functions $U(V^0)$ this can be solved for V^0 in terms of L

$$V^0 = h(L), \quad (3.12)$$

where $h(L)$ is the inverse function of U' . Inserted back into (3.10) we indeed obtain

$$\mathcal{L} = - \int d^2\theta d^2\bar{\theta} K(L) + \mathcal{L}_m, \quad (3.13)$$

where $K(L) = 2h(L)L - U(h(L))$ is the Legendre transform of U . This establishes that (3.10) is the correct starting point for deriving the dual action to which we turn now.

The dual action can be obtained by eliminating Φ_α in favor of V^0 using the equation of motion of Φ_α . More precisely we first replace Φ_α by an unconstrained superfield Y via $\Phi_\alpha = \bar{D}^2 D_\alpha Y$ and then vary the action (3.10) with respect to Y . This yields

$$D^\alpha \left((m^A f_{AB} m^B + \frac{i}{2} e_A m^A) \Phi_\alpha \right) = -\frac{1}{2} D^\alpha \left(W_\alpha^0 + (im^A f_{AB} - \frac{1}{2} e_B) W_\alpha^B \right), \quad (3.14)$$

where $W^0 = -\frac{1}{4} \bar{D}^2 D_\alpha V^0$. The solution of (3.14) reads

$$\Phi_\alpha = -\frac{1}{2} \frac{(im^A f_{AB} - \frac{1}{2} e_B) W_\alpha^B + W_\alpha^0}{m^C f_{CD} m^D + \frac{i}{2} e_C m^C}. \quad (3.15)$$

As promised Φ_α is expressed in terms of W_α^A and W_α^0 which is the field strength of the newly introduced vector multiplet V^0 . Inserted back into (3.10) we obtain the Lagrangian for $n_V + 1$ vector multiplets

$$\mathcal{L} = \frac{1}{4} \int d^2\theta \hat{f}_{\hat{A}\hat{B}} W^{\hat{A}} W^{\hat{B}} + \int d^2\theta d^2\bar{\theta} U(V^0), \quad \hat{A} = 0, \dots, n_V, \quad (3.16)$$

where we have introduced the $(n_V + 1) \times (n_V + 1)$ dimensional gauge coupling matrix $\hat{f}_{\hat{A}\hat{B}}$ given by

$$\begin{aligned} \hat{f}_{AB} &= f_{AB} + \hat{f}_{00}^{-1} \hat{f}_{0A} \hat{f}_{0B} , & \hat{f}_{0A} &= \hat{f}_{00} (im^B f_{AB} - \frac{1}{2} e_A) , \\ \hat{f}_{00} &= [M^2 + iM_T^2]^{-1} = [m^A f_{AB} m^B + \frac{i}{2} e_A m^A]^{-1} . \end{aligned} \quad (3.17)$$

(M and M_T are defined in (2.22).) We immediately see that due to the second term in (3.16) V^0 is a massive vector multiplet. However, we have not yet fixed the original two-form gauge invariance displayed in (2.13) and as a consequence the action (3.16) contains one unphysical vector multiplet. This gauge invariance can be used to gauge away one of the original n_V vector multiplets leaving one massive and $n_V - 1$ massless vector multiplets as the physical spectrum. (Note that V^0 is invariant under the transformation (2.13) and thus cannot be gauged away.)

The fact that one of the vector multiplets in (3.16) is unphysical can also be seen from the gauge coupling matrix $\text{Re } \hat{f}_{\hat{A}\hat{B}}$ which has a zero eigenvalue while the matrix of θ -angles given by $\text{Im } \hat{f}_{\hat{A}\hat{B}}$ has one constant eigenvalue. This follows directly from (3.17) which using (2.22) implies

$$m^A \hat{f}_{AB} = -\frac{i}{2} e_B , \quad m^A \hat{f}_{0A} = i . \quad (3.18)$$

Or in other words $m^A \text{Re } \hat{f}_{AB} = 0$ and hence $\text{Re } \hat{f}_{\hat{A}\hat{B}}$ has the null vector $(0, m^A)$. This implies that one (linear combination) of the vector fields only has a topological coupling but no kinetic term.⁹

Let us give the Lagrangian (3.16) in components. In order to do so we expand V^0 as

$$\begin{aligned} V^0 &= \frac{1}{2} A^0 + \sqrt{2} \theta \psi^0 + \sqrt{2} \bar{\theta} \bar{\psi}^0 - \theta \sigma^m \bar{\theta} v_m^0 + \theta \theta F^0 + \bar{\theta} \bar{\theta} F^{*0} \\ &\quad + i \theta \bar{\theta} \bar{\theta} (\bar{\lambda}^0 + \frac{1}{\sqrt{2}} \bar{\sigma}^m \partial_m \psi^0) - i \bar{\theta} \bar{\theta} \theta (\lambda^0 - \frac{1}{\sqrt{2}} \sigma^m \partial_m \bar{\psi}^0) + \frac{1}{2} \theta \bar{\theta} \bar{\theta} (D^0 + \square A^0) , \end{aligned} \quad (3.19)$$

where A^0 is real. Together with (2.11) we insert (3.19) into (3.16) and arrive at

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} \text{Re } \hat{f}_{\hat{A}\hat{B}} F^{\hat{A}mn} F_{mn}^{\hat{B}} + \frac{1}{8} \text{Im } \hat{f}_{\hat{A}\hat{B}} \epsilon^{mnpq} F_{mn}^{\hat{A}} F_{pq}^{\hat{B}} - \frac{1}{4} U'' v_m^0 v^{0m} \\ &\quad + U'' (F^0 F^{0*} - \frac{1}{2} \partial_m A^0 \partial^m A^0) - \frac{1}{2} U' D^0 - \frac{1}{2} \text{Re } \hat{f}_{\hat{A}\hat{B}} D^{\hat{A}} D^{\hat{B}} \\ &\quad - \frac{i}{2} \left(\hat{f}_{\hat{A}\hat{B}} \lambda^{\hat{A}} \sigma^m \partial_m \bar{\lambda}^{\hat{B}} + \hat{f}_{\hat{A}\hat{B}}^* \bar{\lambda}^{\hat{A}} \bar{\sigma}^m \partial_m \lambda^{\hat{B}} \right) \\ &\quad + \frac{1}{2} U'' (i\sqrt{2}(\psi^0 \lambda^0 - \bar{\psi}^0 \bar{\lambda}^0) - i(\psi^0 \sigma^m \partial_m \bar{\psi}^0 + \bar{\psi}^0 \bar{\sigma}^m \partial_m \psi^0)) + \frac{1}{4} U'''' \psi^0 \psi^0 \bar{\psi}^0 \bar{\psi}^0 \\ &\quad + \frac{1}{2} U''' (-F^{0*} \psi^0 \psi^0 - F^0 \bar{\psi}^0 \bar{\psi}^0 + \bar{\psi}^0 \bar{\sigma}^m \psi^0 v_m^0) + \dots , \end{aligned} \quad (3.20)$$

where again we are neglecting terms proportional to $\partial_i \hat{f}_{\hat{A}\hat{B}}$.

⁹This is again in complete analogy to the situation found in $N = 2$ [11].

The next step is to eliminate the auxiliary fields $D^{\hat{A}}$ and F^0 . F^0 can be straightforwardly eliminated by its equation of motion

$$F^0 = \frac{1}{2} \frac{U'''}{U''} \bar{\psi}^0 \psi^0 . \quad (3.21)$$

For $D^{\hat{A}}$ the situation is more involved since one of the vector multiplets is unphysical and its equation of motion cannot be used.¹⁰ Instead we should only vary with respect to the auxiliary fields of the physical multiplets or in other words first fix the gauge invariance (2.13).

One way to obtain the appropriate constraint is to consider the transformation properties of eq. (3.15) under the transformation (2.4), (2.13). In section 2.1 we showed that the gauge transformation (2.4) can be used to go to a WZ-gauge for Φ . In order to fix the corresponding gauge on the the right hand side of (3.15) we first rewrite (3.15) as

$$\begin{aligned} \Phi_\alpha = & -\frac{1}{2} \frac{M^2}{M^4 + M_T^4} \left(-(m^A \text{Im} f_{AB} + \frac{1}{2} e_B) W_\alpha^B + W_\alpha^0 + \frac{M_T^2}{M^2} m^A \text{Re} f_{AB} W_\alpha^B \right) \\ & - \frac{1}{2} \frac{i M_T^2}{M^4 + M_T^4} \left((m^A \text{Im} f_{AB} + \frac{1}{2} e_B) W_\alpha^B - W_\alpha^0 + \frac{M^2}{M_T^2} m^A \text{Re} f_{AB} W_\alpha^B \right) . \end{aligned} \quad (3.22)$$

The constraint on the D -terms arises from the θ -component of this equation. In the WZ-gauge one has (c.f. (2.7))

$$W^A|_\theta = \delta_\alpha^\beta D^A - \frac{i}{2} (\sigma^m \bar{\sigma}^n)_\alpha^\beta F_{mn}^A , \quad \Phi|_\theta = -\delta_\alpha^\beta \frac{1}{2} C - \frac{1}{4} (\sigma^m \bar{\sigma}^n)_\alpha^\beta B_{mn} . \quad (3.23)$$

Inserted into (3.22) we see that the imaginary part of the right hand has to vanish in the WZ-gauge or in other words the constraint

$$(m^A \text{Im} f_{AB} + \frac{1}{2} e_B) D^B - D^0 + \frac{M^2}{M_T^2} \text{Re} f_{AB} m^A D^B = 0 \quad (3.24)$$

has to be imposed.

Using the constraint (3.24) we can eliminate for example the D^0 field from the Lagrangian (3.20) and are left with the following terms containing D^A

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} U' \left(\frac{M^2}{M_T^2} \text{Re} f_{AB} m^A + m^A \text{Im} f_{AB} + \frac{1}{2} e_B \right) D^B \\ & - \frac{1}{2} \left(\text{Re} f_{AB} + \frac{M^2}{M_T^4} m^C m^D \text{Re} f_{AC} \text{Re} f_{BD} \right) D^A D^B + \dots . \end{aligned} \quad (3.25)$$

From (3.25) we can now determine the equation of motion for D^A as

$$D^A = -\frac{1}{4} U' \text{Re} f^{-1 AC} (e_C + 2 \text{Im} f_{BC} m^B) . \quad (3.26)$$

¹⁰This ‘difficulty’ can also be seen from the fact that $\text{Re} \hat{f}_{\hat{A}\hat{B}}$ has a zero eigenvalue.

Inserting everything back into (3.25) then results in

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{4}\text{Re } f_{\hat{A}\hat{B}} F^{\hat{A}mn} F_{mn}^{\hat{B}} + \frac{1}{8}\text{Im } f_{\hat{A}\hat{B}} \epsilon^{mnpq} F_{mn}^{\hat{A}} F_{pq}^{\hat{B}} - \frac{1}{4}U'' v_m^0 v^{0m} \\
& - \frac{1}{2}U'' \partial_m A^0 \partial^m A^0 - \frac{i}{2} \left(\hat{f}_{\hat{A}\hat{B}} \lambda^{\hat{A}} \sigma^m \partial_m \bar{\lambda}^{\hat{B}} + \hat{f}_{\hat{A}\hat{B}}^* \bar{\lambda}^{\hat{A}} \bar{\sigma}^m \partial_m \lambda^{\hat{B}} \right) \\
& + \frac{1}{2}U'' (i\sqrt{2}(\psi^0 \lambda^0 - \bar{\psi}^0 \bar{\lambda}^0) - i(\psi^0 \sigma^m \partial_m \bar{\psi}^0 + \bar{\psi}^0 \bar{\sigma}^m \partial_m \psi^0)) \\
& + \frac{1}{4} \left(U'''' - \frac{U'''}{U''} \right) \psi^0 \psi^0 \bar{\psi}^0 \bar{\psi}^0 + \frac{1}{2}U''' \bar{\psi}^0 \bar{\sigma}^m \psi^0 v_m^0 - V + \dots,
\end{aligned} \tag{3.27}$$

where the potential is given by

$$\begin{aligned}
V &= \frac{1}{2}\text{Re } f_{AB} D^A D^B + \frac{1}{8}M^2 U'^2 \\
&= \frac{1}{32}U'^2 \left((e_A + 2\text{Im } f_{AC} m^C) \text{Re } f^{-1AB} (e_B + 2\text{Im } f_{BD} m^D) + 4\text{Re } f_{AB} m^A m^B \right).
\end{aligned} \tag{3.28}$$

This potential indeed coincides with the potential (2.21) for $\frac{1}{2}U'(A) = C$. For this identification also the kinetic terms agree which follows from the fact that $U(A)$ and $K(L)$ are related by a Legendre transformation. Note that (3.27) contains apart from the kinetic terms a mass term for the vector v_m^0 and its fermionic partners ψ^0, λ^0 which is proportional to U'' . This ends our discussion of the dual action and we showed that in both formulations the scalar potential agrees.

Let us briefly summarize. Motivated and guided by the results of refs. [11, 13] we proposed an $N = 1$ superfield action (2.9), (2.15) for a massive tensor multiplet coupled to n_V vector and n_c chiral multiplets. We computed the component form of the action and showed that the resulting potential (2.20), (2.21) agrees with the potential obtained in a Kaluza-Klein reduction of type IIB orientifolds. The potential has the ‘unusual’ feature that it is not solely determined by auxiliary fields but has a direct mass term for the scalar in the tensor multiplet. We also discussed the dual superspace action where the massive tensor multiplet has been dualized to a massive vector multiplet. We computed the component form of the dual action and showed that the potential in the two dual formulation coincide.

Acknowledgments

This work is supported by DFG – The German Science Foundation, the European RTN Programs HPRN-CT-2000-00148 and the DAAD – the German Academic Exchange Service.

We have greatly benefited from conversations with Thomas Grimm and Florian Schwennsen.

References

- [1] C. Bachas, “A Way to break supersymmetry,” [hep-th/9503030](#).
- [2] J. Polchinski and A. Strominger, “New Vacua for Type II String Theory,” *Phys. Lett. B* **388** (1996) 736 [[arXiv:hep-th/9510227](#)].
- [3] T. R. Taylor and C. Vafa, “RR flux on Calabi-Yau and partial supersymmetry breaking,” *Phys. Lett. B* **474** (2000) 130 [[arXiv:hep-th/9912152](#)].
- [4] P. Mayr, “On supersymmetry breaking in string theory and its realization in brane worlds,” *Nucl. Phys. B* **593** (2001) 99 [[arXiv:hep-th/0003198](#)].
- [5] S. B. Giddings, S. Kachru and J. Polchinski, “Hierarchies from fluxes in string compactifications,” *Phys. Rev. D* **66** (2002) 106006 [[arXiv:hep-th/0105097](#)].
- [6] G. Curio, A. Klemm, D. Lüst and S. Theisen, “On the vacuum structure of type II string compactifications on Calabi-Yau spaces with H-fluxes,” *Nucl. Phys. B* **609** (2001) 3 [[arXiv:hep-th/0012213](#)];
G. Curio, A. Klemm, B. Körs and D. Lüst, “Fluxes in heterotic and type II string compactifications,” *Nucl. Phys. B* **620** (2002) 237 [[arXiv:hep-th/0106155](#)].
- [7] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, “De Sitter vacua in string theory,” *Phys. Rev. D* **68**.
- [8] C. P. Burgess, R. Kallosh and F. Quevedo, “de Sitter string vacua from supersymmetric D-terms,” *JHEP* **0310**, 056 (2003) [[arXiv:hep-th/0309187](#)].
- [9] L. J. Romans, “Massive N=2a Supergravity In Ten-Dimensions,” *Phys. Lett. B* **169**, 374 (1986).
- [10] M. Haack, J. Louis and H. Singh, “Massive type IIA theory on K3,” *JHEP* **0104**, 040 (2001) [[arXiv:hep-th/0102110](#)].
- [11] J. Louis and A. Micu, “Type II theories compactified on Calabi-Yau three-folds in the presence of background fluxes,” *Nucl. Phys. B* **635** (2002) 395 [[arXiv:hep-th/0202168](#)].
- [12] G. Dall’Agata, R. D’Auria, L. Sommovigo and S. Vaula, “D = 4, N = 2 gauged supergravity in the presence of tensor multiplets,” *Nucl. Phys. B* **682**, 243 (2004) [[arXiv:hep-th/0312210](#)];
L. Sommovigo and S. Vaula, “D = 4, N = 2 supergravity with Abelian electric and magnetic charge,” [arXiv:hep-th/0407205](#);
R. D’Auria, L. Sommovigo and S. Vaula, “N = 2 supergravity Lagrangian coupled to tensor multiplets with electric and magnetic fluxes,” [arXiv:hep-th/0409097](#).
- [13] T. W. Grimm and J. Louis, “The effective action of N = 1 Calabi-Yau orientifolds,” [arXiv:hep-th/0403067](#).
- [14] W. Siegel, “Gauge Spinor Superfield As A Scalar Multiplet,” *Phys. Lett. B* **85** (1979) 333.

- [15] S.J. Gates, M.T. Grisaru, M. Rocek, W. Siegel, “Superspace”, Benjamin/Cummings, 1983.
- [16] I.L. Buchbinder, S.M. Kuzenko, “Ideas and methods of supersymmetry and supergravity,” Inst. Phys. 1998.
- [17] R. D’Auria and S. Ferrara, “Dyonic Masses from Conformal Field Strengths in D even Dimensions,” arXiv:hep-th/0410051.
- [18] B. K rs and P. Nath, “A supersymmetric Stueckelberg U(1) extension of the MSSM,” arXiv:hep-ph/0406167.
- [19] S. J. Gates, “Super P Form Gauge Superfields,” Nucl. Phys. B **184** (1981) 381.
- [20] U. Lindstrom and M. Rocek, “Scalar Tensor Duality And N=1, N=2 Nonlinear Sigma Models,” Nucl. Phys. B **222** (1983) 285.
- [21] J. P. Derendinger, F. Quevedo and M. Quiros, “The Linear multiplet and quantum four-dimensional string effective actions,” Nucl. Phys. B **428**, 282 (1994) [arXiv:hep-th/9402007].
- [22] P. Binetr y, G. Girardi and R. Grimm, “Linear supermultiplets and nonholomorphic gauge coupling functions,” Phys. Lett. B **265** (1991) 111;
P. Adamietz, P. Binetr y, G. Girardi and R. Grimm, “Supergravity and matter: Linear multiplet couplings and Kahler anomaly cancellation,” Nucl. Phys. B **401** (1993) 257;
P. Binetr y, G. Girardi and R. Grimm, “Supergravity couplings: A geometric formulation,” Phys. Rept. **343**, 255 (2001) [arXiv:hep-th/0005225].
- [23] W. Schulgin, “ $N = 1$ Massive Tensor Multiplets”, Diploma Thesis, Universit t Hamburg, <http://www.desy.de/uni-th/stringth/Works/arbeiten.html>.
- [24] J. Wess, J. Bagger, ”Supersymmetry and Supergravity,” Princeton University Press, Princeton, New Jersey, 1992.
- [25] For a review see, for example, H. Ruegg and M. Ruiz-Altaba, “The Stueckelberg field,” Int. J. Mod. Phys. A **19** (2004) 3265 [arXiv:hep-th/0304245].
- [26] L. Andrianopoli, S. Ferrara and M. A. Lledo, “Axion gauge symmetries and generalized Chern-Simons terms in $N = 1$ JHEP **0404** (2004) 005 [arXiv:hep-th/0402142].